ELSEVIER

Journal of Geometry and Physics 46 (2003) 151–158

JOURNAL OF GEOMETRY AND PHYSICS

www.elsevier.com/locate/jgp

Transitive Lie algebroids of rank 1 and locally conformal symplectic structures

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Received 3 April 2002; received in revised form 21 May 2002

Abstract

The aim of the paper is the study of transitive Lie algebroids with the trivial 1-rank adjoint bundle of isotropy Lie algebras $\boldsymbol{g} \cong M \times \mathbb{R}$. We show that a locally conformal symplectic (l.c.s.) structure defines such a Lie algebroid, so our algebroids are a natural generalisation of l.c.s. structures. We prove that such a Lie algebroid has the Poincaré duality property for the Lie algebroid cohomology (TUIO-property) if and only if the top-dimensional cohomology space is non-trivial. Moreover, if an algebroid is defined by an l.c.s. structure, then this algebroid is a TUIO-Lie algebroid if and only if the associated l.c.s. structure is a globally conformal symplectic structure.

MSC: 58H05; 22A30; 35F05; 37J05; 53C12; 55M05; 57R30

JGP SC: Differential geometry

Keywords: Lie algebroids; Symplectic geometry

1. Introduction

Lie algebroids play an important role in geometry. They appear as naturally associated to many well-known and important geometrical objects like G-principal fibre bundles,

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PII: S0393-0440(02)00128-6

TC-foliations, Poisson and Jacobi manifolds or singular foliations, cf. [1,10–12,17,18,20,21]. Moreover, a Lie algebroid is naturally associated to any Lie pseudogroup. The cohomology ring defined by a Lie algebroid is a suitable setting for characteristic classes of such a geometrical structure—it generalises the classical theory of characteristic classes, cf. [2,3,11,13,16]. Therefore, we think that, it is important to study the properties of this cohomology ring, in particular the Poincaré duality property. In this respect, the second author proved the following theorem, [15].

Theorem 1.1. *Let $A = (A, [\![\cdot,\cdot]\!], \gamma)$ be a transitive Lie algebroid on a connected oriented m-manifold M and let*

$$0 \to \boldsymbol{g} \to A \xrightarrow{\gamma} TM \to 0. \tag{1.1}$$

be the Atiyah sequence of A. Assume that

(a) *the isotropy Lie algebras $\boldsymbol{g}_{|x}$ are unimodular,*
(b) *there exists a non-singular cross-section $\varepsilon \in \operatorname{Sec} \bigwedge^n \boldsymbol{g}$, $n = \operatorname{rank} \boldsymbol{g}$, invariant with respect to the adjoint representation,*

then the family of bilinear homomorphisms

$$P^k : H^k_A(M) \times H^{n+m-k}_{A,c}(M) \to \mathbb{R}, \qquad ([\Phi],[\Psi]) \mapsto \int_M ⨍_{(A,\varepsilon)} \Phi \wedge \Psi$$

$(k = 0, 1, \ldots, n+m)$ yields a Poincaré duality $P : H_A(M) \times H_{A,c}(M) \to \mathbb{R}$ (for the fibre integral $⨍_{(A,\varepsilon)}$, see [14]). In particular, if M is, in addition, compact, then the cohomology algebra $H^*_A(M)$ of real A-differential forms is finite-dimensional and satisfies the Poincaré duality.

The transitive Lie algebroid fulfilling properties (a) and (b) from the above theorem is called a *TUIO-Lie algebroid* [14] (*transitive unimodular invariantly oriented*). For examples of TUIO-Lie algebroids of some principal bundles, non-closed Lie subgroups and TP-foliations see [15].

The second author discovered a very important fact that for the Lie algebroid $A = A(M, \mathcal{F})$ associated to a TC-foliation $\mathcal{F}$ on a manifold M, there is an isomorphism of differential graded algebras $\Omega(A(M, \mathcal{F})) \cong \Omega_b(M, \mathcal{F})$ [15, Theorem 6.10]. Therefore, the cohomology algebra $H_{A(M,\mathcal{F})}(W)$ of the Lie algebroid $A(M, \mathcal{F})$ satisfies the Poincaré duality if and only if the basic cohomology of the foliated manifold $(M, \mathcal{F})$ satisfies the Poincaré duality (W is the basic manifold of $(M, \mathcal{F})$). Additionally, in compact orientable case for $q = \operatorname{codim} \mathcal{F} H^q(M/\mathcal{F}) = H^q_{A(M,\mathcal{F})}(W) = 0$ or $\mathbb{R}$.

The Poincaré property is a very important one for a transversely orientable Riemannian foliation $\mathcal{F}$ on a compact manifold M as the following conditions are equivalent [4,9,19,22]:

(1) the basic cohomology $H(M/\mathcal{F})$ satisfies the Poincaré duality;
(2) $H^q(M/\mathcal{F}) \neq 0$;
(3) the foliation $\mathcal{F}$ is minimalizable.

It is well known that on the total space B of the principal bundle $B(M, SO(q); \mathcal{F})$ of orthonormal frames of the normal bundle of $\mathcal{F}$, there exists a foliation $\mathcal{F}_1$ whose leaves are holonomy coverings of leaves of $\mathcal{F}$. Moreover, this foliation is transversely parallelisable. Its codimension is equal to $q + q(q-1)/2$ and $H^q(M,\mathcal{F}) = H^{q+q(q-1)/2}(B,\mathcal{F})$, cf. [5]. Let $A_{\mathcal{F}_1}$ be the Lie algebroid of the foliation $\mathcal{F}_1$. Combining the above results we have the following equivalence:

- The foliation $\mathcal{F}$ is minimalisable if and only if $A_{\mathcal{F}_1}$ is a TUIO-Lie algebroid.

The epimorphy of the fibre integral $\int_{(A,\varepsilon)} : \Omega^*_{(A,\varepsilon)}(M) \to \Omega^{*-n}(M)$ [14, Prop. 4.2.1(e)] easily gives the following implication [15] (by assuming the compactness and the orientability of M):

$$A \text{ is a TUIO-Lie algebroid} \Rightarrow H_A^{n+m}(M) \neq 0. \tag{1.2}$$

(Remark: Theorem 1.1 yields more: $H_A^{n+m}(M) = \mathbb{R}$).

Open question: Can the implication in (1.2) be changed into the equivalence?

The paper is dedicated to the study of transitive Lie algebroids with the trivial 1-rank adjoint bundle of isotropy Lie algebras $\boldsymbol{g} \cong M \times \mathbb{R}$. We discover that a locally conformal symplectic (l.c.s.) structure defines such a Lie algebroid. We prove that this algebroid is a TUIO-Lie algebroid if and only if the associated l.c.s. structure is a globally conformal symplectic structure.

2. Main results

We recall [11,14] that a connection $\lambda : TM \to A$ in a transitive Lie algebroid A determines a covariant derivative ∇ in the adjoint bundle $\boldsymbol{g}$ by the formula: $\nabla_X \nu = [\![\lambda X, \nu]\!]$, $\nu \in \operatorname{Sec} \boldsymbol{g}$. If the structural Lie algebras $\boldsymbol{g}_{|x}$ are abelian, then to all connections λ there corresponds the same covariant derivative ∇ in $\boldsymbol{g}$, called then *canonical* or *characteristic*, and ∇ is flat.

Each flat covariant derivative in $\boldsymbol{g} = M \times \mathbb{R}$ is of the form

$$\nabla_X f = \partial_X f + \omega(X) \cdot f,$$

where ω is a closed 1-differentiable form on M. Such closed one forms define new cohomology operators [6,8].

According to the general structure theorem [10,17], we obtain:

- Each transitive Lie algebroid on M with a trivial adjoint bundle $\boldsymbol{g} \cong M \times \mathbb{R}$ is isomorphic to

  $$A = (M \times \mathbb{R}) \oplus TM \tag{2.1}$$

  with $\gamma = \mathrm{pr}_2 : (M \times \mathbb{R} \oplus TM \to TM$ as the anchor and the bracket $[\![\cdot,\cdot]\!]$ in $\operatorname{Sec} A$ defined via some flat covariant derivative ∇ in $M \times \mathbb{R}$ and a 2-form $\Omega \in \Omega^2(M)$ fulfilling the Bianchi identity $\nabla\Omega = 0$ in the following way:

  $$[\![(f,X),(g,Y)]\!] = (-\Omega(X,Y) + \nabla_X g - \nabla_Y f, [X,Y]).$$

  The flat covariant derivative ∇ is then characteristic.

Therefore, the condition $\nabla\Omega = 0$ is equivalent to $\mathrm{d}\Omega = -\omega \wedge \Omega$. Hence a transitive Lie algebroid with trivial adjoint bundle is determined by the following data: a closed 1-form ω and a 2-form Ω such that $\mathrm{d}\Omega = -\omega \wedge \Omega$.

According to [6], the pair $(-\omega, \Omega)$ determining Lie algebroid (2.1) is precisely an l.c.s. structure on our manifold provided that the 2-form Ω is non-degenerate. Therefore our transitive algebroid of rank 1 are a natural generalisation of the l.c.s. structure. When the 1-form ω is exact the structure is called globally conformal symplectic. For more information about these structures, see [6–8,23].

If α is a closed 1-form on M, it defines an operator (see [6,8])

$$d^\alpha : \Omega^*(M) \to \Omega^{*+1}(M), \qquad \gamma \mapsto \mathrm{d}\gamma + \alpha \wedge \gamma.$$

Obviously $d^\alpha d^\alpha = 0$ and the corresponding cohomology we denote by $H^*_\alpha(M)$.

- If two forms α and α' are cohomologous in $H^1_{\mathrm{DR}}(M)$, then $H^*_\alpha(M)$ is isomorphic to $H^*_{\alpha'}(M)$.

For completeness, we show this briefly. If $\alpha' = \alpha + \mathrm{d}\varphi/\varphi$, $\varphi > 0$, then the linear isomorphism on cycles $Z_\alpha \to Z_{\alpha'}$, $\gamma \mapsto \gamma/\varphi$, transforms d^α-exact forms into $d^{\alpha'}$-exact forms giving an isomorphism on cohomology. Notice that, the inverse homomorphism is given by $Z_{\alpha'} \to Z_\alpha$, $\gamma' \mapsto \varphi\gamma' = \gamma'/(1/\varphi)$, and $\alpha = \alpha' - \mathrm{d}\varphi/\varphi = \alpha' + \mathrm{d}(1/\varphi)/(1/\varphi)$. In particular if α is an exact form, then $H^*_\alpha(M) \cong H^*_{\mathrm{DR}}(M)$.

The second fundamental theorem on d^α-cohomology is the following (it was discovered by Guedira and Lichnerowicz [8], for a short elementary proof see [6]).

Theorem 2.1. *If α is not exact (M is connected and orientable, $\dim M = m$, but M can be compact or not), then $H^m_\alpha(M) = 0$, i.e. the following differential equation:*

$$\Delta = \mathrm{d}\gamma + \alpha \wedge \gamma$$

has a global solution $\gamma \in \Omega^{m-1}(M)$ for each m-form $\Delta \in \Omega^m(M)$.

With this notation in mind, we prove the following theorem.

Theorem 2.2. *Let M be a connected orientable m-manifold. If ∇ is a flat covariant derivative in $\mathbf{g} = M \times \mathbb{R}$ and $\nabla_X f = \partial_X f + \omega(X)\cdot f$, for a closed 1-differential form $\omega \in \Omega^1(M)$, then*

(a) *the Lie algebroid A, see (2.1), is a TUIO-Lie algebroid if and only if the form ω is exact, in particular, if $H^1(M) = 0$, then A is a TUIO-Lie algebroid,*
(b) $H^{1+m}_A(M) = H^m_{-\omega}(M)$,
(c) $H^{1+m}_{A,c}(M) = H^m_{-\omega,c}(M)$.

The above theorem permits us to demonstrate our main result, i.e. the following theorem.

Theorem 2.3. *Let M be a compact orientable m-manifold and let A be a transitive Lie algebroid with trivial 1-rank adjoint bundle of isotropy Lie algebras $\mathbf{g} \cong M \times \mathbb{R}$. Then A is a TUIO-Lie algebroid if and only if $H^{1+m}_A(M) \neq 0$.*

Proof. We have only to demonstrate that the condition $H_A^{1+m}(M) \neq 0$ implies that A is a TUIO-Lie algebroid. If $H_A^{1+m}(M) \neq 0$. Theorem 2.2 ensures that $H_{-\omega}^m(M) \neq 0$. In that case the 1-form $-\omega$ must be exact as for non-exact 1-forms $\alpha H_\alpha^m(M) = 0$, cf. [6,8]. So according to Theorem 2.2 as the form ω is exact, the Lie algebroid A is a TUIO-Lie algebroid. □

For local conformally symplectic structures we have the following corollary.

Corollary 2.1. *Let (ω, Ω) be an l.c.s. structure on M. Then the associated transitive Lie algebroid of rank* 1 *is a TUIO-algebroid if and only if the l.c.s. structure is a global conformal symplectic structure.*

3. Proof of Theorem 2.2

First we have to show that the transitive Lie algebroid (2.1) described above is a TUIO-Lie algebroid if and only if ω is exact. If f is a d_A-invariant, then f is ∇-constant and, by assuming that f is non-singular, we obtain $\omega = \mathrm{d}(-\ln(f))$. Conversely, if $\omega = \mathrm{d}(\tilde{f})$, then $f = e^{-\tilde{f}}$ is a non-singular $a\mathrm{d}_A$-invariant cross-section of $\boldsymbol{g} = M \times \mathbb{R}$.

Denote by $\mathrm{pr}_1 : (M \times \mathbb{R}) \oplus TM \to M \times \mathbb{R}$ the projection onto the first factor. Let d_A be the exterior derivative of A-differential forms $\Omega_A(M) = \mathrm{Sec} \bigwedge A^\star$. For a homomorphism of Lie algebroids $F : A' \to A$ over $t : M' \to M$, the pullback of differential forms $F^\star : \Omega_A(M) \to \Omega_{A'}(M')$ is defined by $(F^\star \Psi)_{|x}(v_1, \dots, v_k) = \Psi_{|tx}(Fv_1, \dots, Fv_k)$ and the following equalities $F^\star(\Psi \wedge \Phi) = F^\star\Psi \wedge F^\star\Phi$, $F^\star(\mathrm{d}_A\Psi) = \mathrm{d}_{A'}(\Psi)$ hold.

Represent the algebra of A-differential forms for $A = (M \times \mathbb{R} \oplus TM$ as the skew tensor product of the anticommutative graded algebras

$$\Omega_A(M) = \mathrm{Sec}\left(\bigwedge(M \times \mathbb{R})^\star \otimes \bigwedge T^\star M\right) = \bigwedge \mathbb{R}^\star \otimes \Omega(M),$$

where $\bigwedge \mathbb{R}^\star$ is the exterior algebra over a one-dimensional graded vector space (homogeneous of degree 1). Therefore, each $(k+1)$-form $\Phi^{k+1} \in \Omega_A^{k+1}(M)$ has the unique representation of the form:

$$\Phi^{k+1} = 1 \otimes \varphi^{k+1} + \mathbf{1}^\star \otimes \varphi^k, \tag{3.1}$$

where φ^k and φ^{k+1} are differential forms on M of degrees k and $k+1$, respectively, and $\mathbf{1}^\star = \mathrm{id}_\mathbb{R}$. Clearly, $1 \otimes \varphi^{k+1} = \gamma^\star \varphi^{k+1}$ and $\mathbf{1}^\star \otimes \varphi^k = \mathrm{pr}_1^\star(\varepsilon^\star) \wedge \gamma^\star \varphi^k$ for the cross-section $\varepsilon^\star$ of $M \times \mathbb{R}^\star$ defined by $\varepsilon^\star(x) \equiv \mathbf{1}^\star$. The structure of the skew tensor product of the anticommutative graded algebras $\bigwedge \mathbb{R}^\star$ and $\Omega(M)$ is given by

$$\begin{aligned}&(1 \otimes \varphi^{k+1} + \mathbf{1}^\star \otimes \varphi^k) \wedge (1 \otimes \varphi^{l+1} + \mathbf{1}^\star \otimes \varphi^l)\\ &\quad = 1 \otimes (\varphi^{k+1} \wedge \psi^{l+1}) + \mathbf{1}^\star \otimes (\varphi^k \wedge \psi^{l+1} + (-1)^{k+1}\varphi^{k+1} \wedge \psi^l).\end{aligned}$$

Via representation (3.1), we easily show that the differential d_A in $\bigwedge \mathbb{R}^\star \otimes \Omega(M)$ is given by

$$\mathrm{d}_A(1 \otimes \varphi^k + \mathbf{1}^\star \otimes \varphi^{k-1}) = 1 \otimes (\mathrm{d}\varphi^k + \Omega \wedge \varphi^{k-1}) + \mathbf{1}^\star \otimes (\omega \wedge \varphi^{k-1} - \mathrm{d}\varphi^{k-1}).$$

To see this, we have, at first,

$$\mathrm{d}_A(1\otimes\varphi^k)=\mathrm{d}_A(\gamma^\star\varphi^k)=\gamma^\star(\mathrm{d}\varphi^k)=1\otimes\mathrm{d}\varphi^k$$

and

$$\mathrm{d}_A(\mathbf{1}^\star\otimes 1)=\mathrm{d}_A(\mathrm{pr}_1^\star(\varepsilon^\star))=1\otimes\Omega+\mathbf{1}^\star\otimes\omega.$$

Indeed,

$$\begin{aligned}&\mathrm{d}_A(\mathrm{pr}_1^\star(\varepsilon^\star))((f,X),(g,Y))\\&\quad=\partial_X g-\partial_Y f-\mathrm{pr}_1^\star(\varepsilon^\star)([\![(f,X),(g,Y)]\!])\\&\quad=\partial_X g-\partial_Y f-\mathrm{pr}_1^\star(\varepsilon^\star)(-\Omega(X,Y)+\nabla_X g-\nabla_Y f,[X,Y])\\&\quad=\partial_X g-\partial_Y f-(-\Omega(X,Y)+\nabla_X g-\nabla_Y f)\\&\quad=\partial_X g-\partial_Y f+\Omega(X,Y)-(\partial_X g+\omega(X)\cdot g)+\partial_Y f+\omega(Y)\cdot f\\&\quad=\Omega(X,Y)-\omega(X)\cdot g+\omega(Y)\cdot f\\&\quad=\Omega(X,Y)+\mathrm{pr}_1^\star(\varepsilon^\star)\wedge\mathrm{pr}_1^\star(\omega)((f,X),(g,Y))\\&\quad=(1\otimes\Omega+\mathbf{1}^\star\otimes\omega)((f,X),(g,Y)).\end{aligned}$$

So

$$\begin{aligned}&\mathrm{d}_A(1\otimes\varphi^k+1^\star\otimes\varphi^{k-1})\\&\quad=\mathrm{d}_A(1\otimes\varphi^k)+\mathrm{d}_A((1^\star\otimes 1)\wedge(1\otimes\varphi^{k-1}))\\&\quad=1\otimes\mathrm{d}\varphi^k+\mathrm{d}_A(\mathbf{1}^\star\otimes 1)\wedge(1\otimes\varphi^{k-1})-(\mathbf{1}^\star\otimes 1)\wedge\mathrm{d}_A(1\otimes\varphi^{k-1})\\&\quad=1\otimes\mathrm{d}\varphi^k+(1\otimes\Omega+\mathbf{1}^\star\otimes\omega)\wedge(1\otimes\varphi^{k-1})-(\mathbf{1}^\star\otimes 1)\wedge(1\otimes\mathrm{d}\varphi^{k-1})\\&\quad=1\otimes(\mathrm{d}\varphi^k+\Omega\wedge\varphi^{k-1})+\mathbf{1}^\star\otimes(\omega\wedge\varphi^{k-1})-\mathbf{1}^\star\otimes\mathrm{d}\varphi^{k-1}\\&\quad=1\otimes(\mathrm{d}\varphi^k+\Omega\wedge\varphi^{k-1})+\mathbf{1}^\star\otimes(\omega\wedge\varphi^{k-1}-\mathrm{d}\varphi^{k-1}).\end{aligned}$$

We look at the graded algebra $\bigwedge\mathbb{R}^\star\otimes\Omega(M)$ as follows:

$$\begin{aligned}&\textstyle\bigwedge\mathbb{R}^\star\otimes\Omega(M)=(\mathbb{R}\otimes\Omega^\star(M))\oplus(\mathbb{R}^\star\otimes\Omega^{\star-1}(M))\equiv\Omega^\star(M)\oplus\Omega^{\star-1}(M),\\&1\otimes\varphi^k+\mathbf{1}^\star\otimes\varphi^{k-1}\mapsto(\varphi^k,\varphi^{k-1}),\\&(\varphi^k,\varphi^{k-1})\wedge(\varphi^k,\psi^{k-1})=(\varphi^{k+1}\wedge\psi^{l+1},\varphi^k\wedge\psi^{l+1}+(-1)^{k+1}\varphi^{k+1}\wedge\psi^l).\end{aligned}$$

Therefore the exterior derivative on the level of k-forms is then given by the formula

$$\begin{aligned}&\mathrm{d}_A^k:\Omega^k(M)\oplus\Omega^{k-1}(M)\to\Omega^{k+1}(M)\oplus\Omega^k(M),\\&\mathrm{d}_A(\varphi^k,\varphi^{k-1})=(\mathrm{d}\varphi^k+\Omega\wedge\varphi^{k-1},\omega\wedge\varphi^{k-1}-\mathrm{d}\varphi^{k-1}).\end{aligned}$$

In particular, $\mathrm{d}_A(\varphi^k,\varphi^{k-1})=0$ if and only if $\mathrm{d}\varphi^k-\Omega\wedge\varphi^{k-1}$ and $\mathrm{d}\varphi^{k-1}=\omega\wedge\varphi^{k-1}$.

Let $m=\dim M$. The maximal degree of A-differential forms is $m+1$:

$$\Omega_A^{m+1}\equiv\Omega^{m+1}(M)\otimes\Omega^m(M)=0\otimes\Omega^m(M)=\Omega^m(M)$$

and

$$\mathrm{d}_A^m : \Omega_A^m(M) \equiv \Omega^m(M) \oplus \Omega^{m-1}(M) \to \Omega^m(M) \equiv \Omega_A^{m+1}(M),$$
$$(\varphi^m, \varphi^{m-1}) \mapsto \omega \wedge \varphi^{m-1} - \mathrm{d}\varphi^{m-1}.$$

Put

$$\bar{d}^{k-1} : \Omega^{k-1}(M) \to \Omega^k(M), \qquad \varphi^{k-1} \mapsto \omega \wedge \varphi^{k-1} - \mathrm{d}\varphi^{k-1}.$$

Clearly $\bar{d} = -d^{-\omega}$ and $\bar{d} \circ \bar{d} = 0$,

$$H_A^{m+1}(M) = H^m(\Omega(M), \bar{d}) = H_{-\omega}^m(\Omega(M), \bar{d})$$

and $H_A^{m+1}(M)$ does not depend on the 2-form Ω.

Analogously

$$H_{A,c}^{m+1}(M) = H_c^m(\Omega(M), \bar{d}) = H_{-\omega,c}^m(\Omega(M), \bar{d})$$

which ends the proof.

Therefore, $H_A^{m+1}(M) = 0$ if and only if, for each m-form $\Delta \in \Omega^m(M)$, there exists an $m-1$-form $\varphi \in \Omega^{m-1}(M)$ such that $\Delta = \omega \wedge \varphi - \mathrm{d}\varphi$.

Acknowledgements

Research work partially supported by KBN Grant PB 173/PO/97/13.

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